



# Fractional Derivative Filter for Image Contrast Enhancement with Order Prediction

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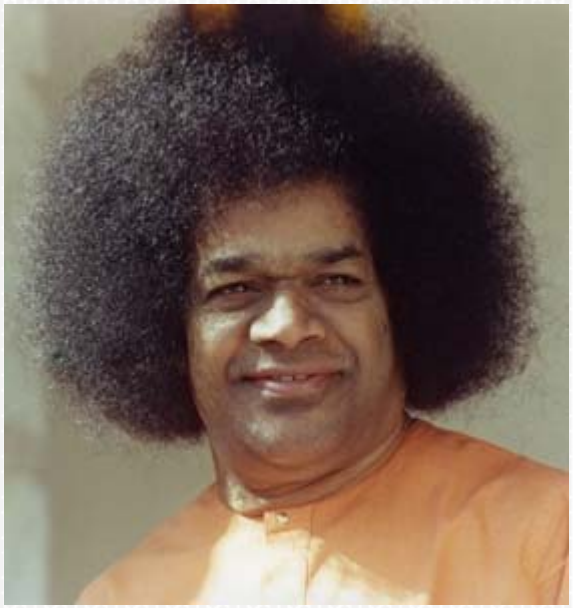
# PRANAMS AT HIS LOTUS FEET



# FRACTIONAL DERIVATIVES

- *Leibniz* :"Can the meaning of derivatives with integer order be generalised to derivatives with non-integer order?"
- *L'Hospital* : "What if n=1/2?"
- *Leibniz* : "Thus it follows that $d^{1/2}x$ will be equal to $x\sqrt{dx:x}$, an apparent paradox, from which one day useful consequences will be drawn."[1]

[1] Letter from Hanover, Germany, September 30,1695. *Leibnizen Mathematische Schriften*, Vol. 2, pp. 301-302. Olms Verlag., Germany, 1962.

# DEFINITIONS*

1730 – L. Euler - "If n is a positive integer, $d^n$ can be found by continued differentiation. Such a way, however is not evident if n is a fraction. But the matter may be expedited with the help of the interpolation of series .."

...

1819 - S. F. LaCroix - $n^{th}$ derivative of $v^m$ by induction $\frac{d^{1/2}}{dv^{1/2}}v = \frac{2\sqrt{v}}{\sqrt{\Pi}}$

...

1835 – J. Liouville – fractional derivative as infinite series $\frac{d^u y}{dx^u} = \sum A_m e^{mx} m^u$

...

1847 – B. Riemann – generalisation of Taylor's series expansion to derive the expression for fractional integration $\frac{d^{-r}}{dx^{-r}}u(x) = \frac{1}{\Gamma(r)}\int_c^x (x-k)^{r-1}u(k)dk$

...

1867-68 – A.K. Grunwald & A.V. Letnikov $D^r f(x) = \lim_{h\to 0} \frac{\sum_{0\le m<\infty} (-1)^m \ {}^rC_m f(x+(r-m)h)}{h^r}$

...

1966-67 – M.Caputo $ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau$

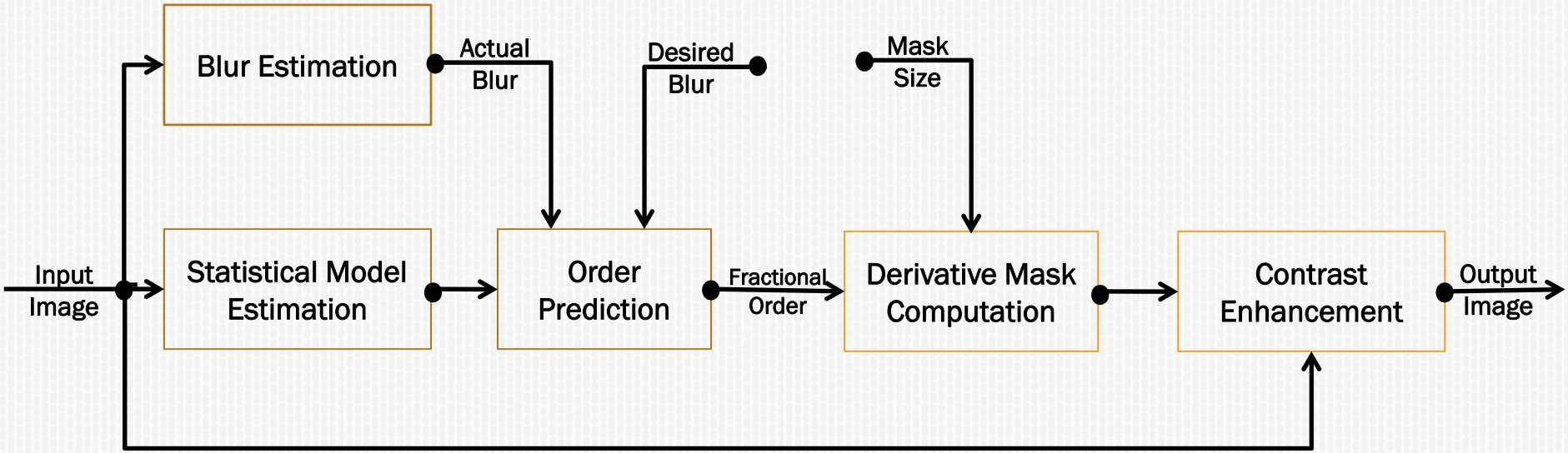
*K.B.Oldham, J.Spanier, "The Fractional Calculus", Dover Publications, NewYork.

# POTENTIAL APPLICATIONS

- Modelling rugged surface of a malignant breast cell nucleus[1]
- Utilising Fractional order PID controller in industrial control systems
- Employing fractional discriminant functions to improve perceptual quality of image[2]
- Fractional Calculus based image and signal processing[3]

[1]Borredon L, Henry B, and Wearne S. "Differentiating the non-differentiable – fractional calculus", Parabola, 35(2):9–19, 1999.
[2]S. Hungenahally, "Neural basis for the design of fractional discriminant functions," Proc. IEEE International Conference on Neural Networks, vol. 1, pp. 1115, 1995.
[3]Yifei Pu, Wang WeiXing, Zhou JiLiu, Wang YiYang, Jia HuaDing, "Fractional differential approach to detecting textural features of digital image and its fractional differential filter implementation," Science in China Series –F –Inf. Sci., vol. 51, No. 9, pp. 1319-1339, 2008.

# PROPOSED SCHEME



# DESIGN OF FRACTIONAL ORDER FILTER

Typical 5x5 FOD Mask

| | | | | |
|---|---|---|---|---|
| G2 | H1 | A2 | B1 | C2 |
| F1 | G1 | A1 | C1 | D1 |
| E2 | E1 | P | E1 | E2 |
| D1 | C1 | A1 | G1 | F1 |
| C2 | B1 | A2 | H1 | G2 |

Result for r=0.34

| | | | | |
|---|---|---|---|---|
| -0.0215 | -0.0706 | -0.0242 | -0.0706 | -0.0215 |
| -0.0706 | -0.0825 | -0.0928 | -0.0825 | -0.0706 |
| -0.0242 | -0.0928 | 2.7304 | -0.0928 | -0.0242 |
| -0.0706 | -0.0825 | -0.0928 | -0.0825 | -0.0706 |
| -0.0215 | -0.0706 | -0.0242 | -0.0706 | -0.0215 |

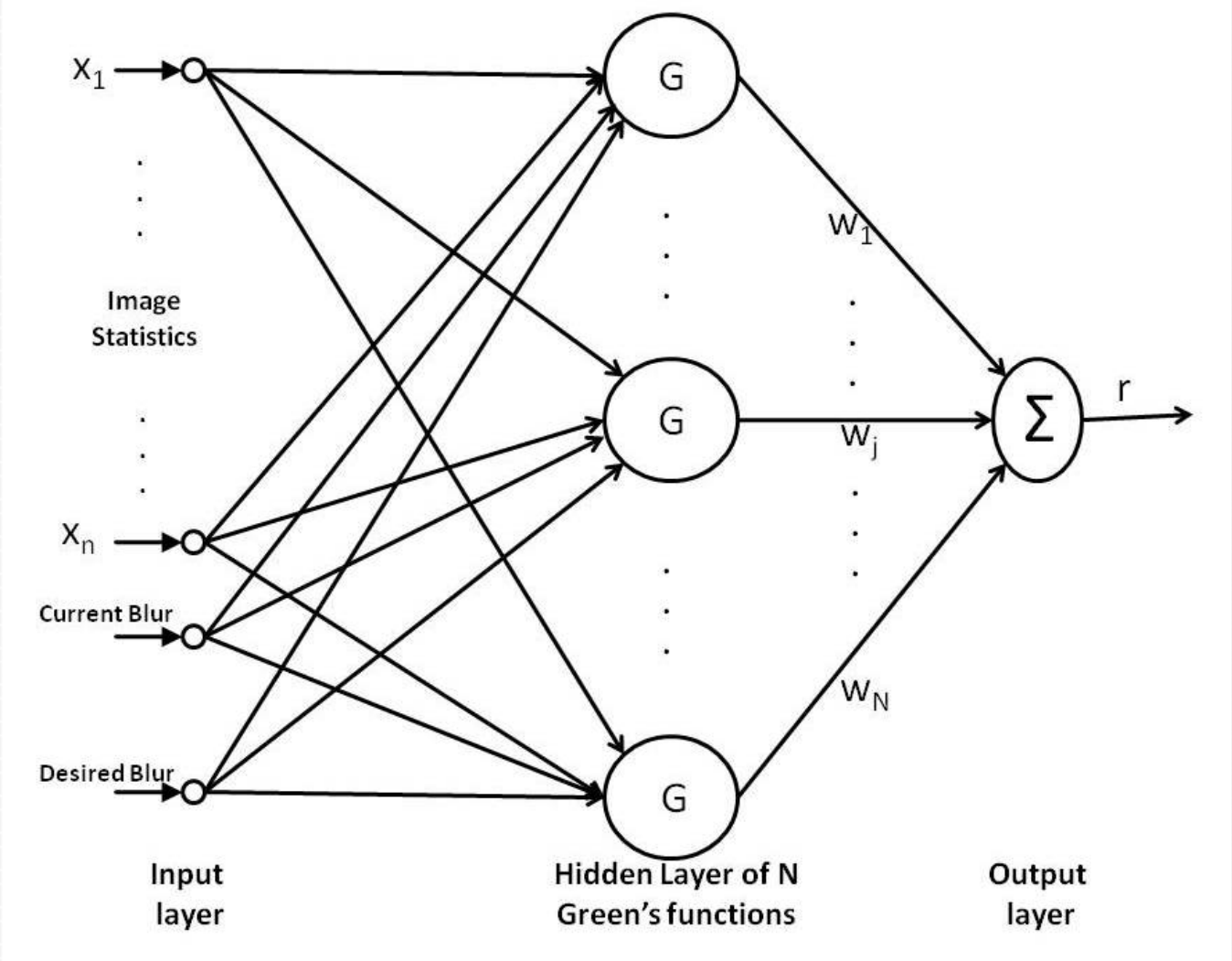
- **Compute h** – the inter-pixel distances, say, between P and D1
- **Use it in the GL** formulation for a **specified** fractional order 'r'

$$D^r f(x) = \underset{h \to 0}{lt} \frac{\sum_{0 \le m < \infty} (-1)^m C_m^r f(x + (r-m)h)}{h^r}$$

# CHALLENGE AND PROPOSED APPROACH

- The challenge here is to predict the fractional order to achieve the desired performance of contrast improvement / blur reduction.
- Radial Basis Function networks with Greens function as processing element have shown to be best multi-dimensional interpolators.
- Therefore, RBF network is used to learn the values of r from an ensemble of training images and their performance over a range of fractional order filters.

# PREDICTION ARCHITECTURE



# ALGORITHM

## Aim

- Given a test image, based on its statistics, initial blur and the desired blur value, we should be able to predict the appropriate fractional order.

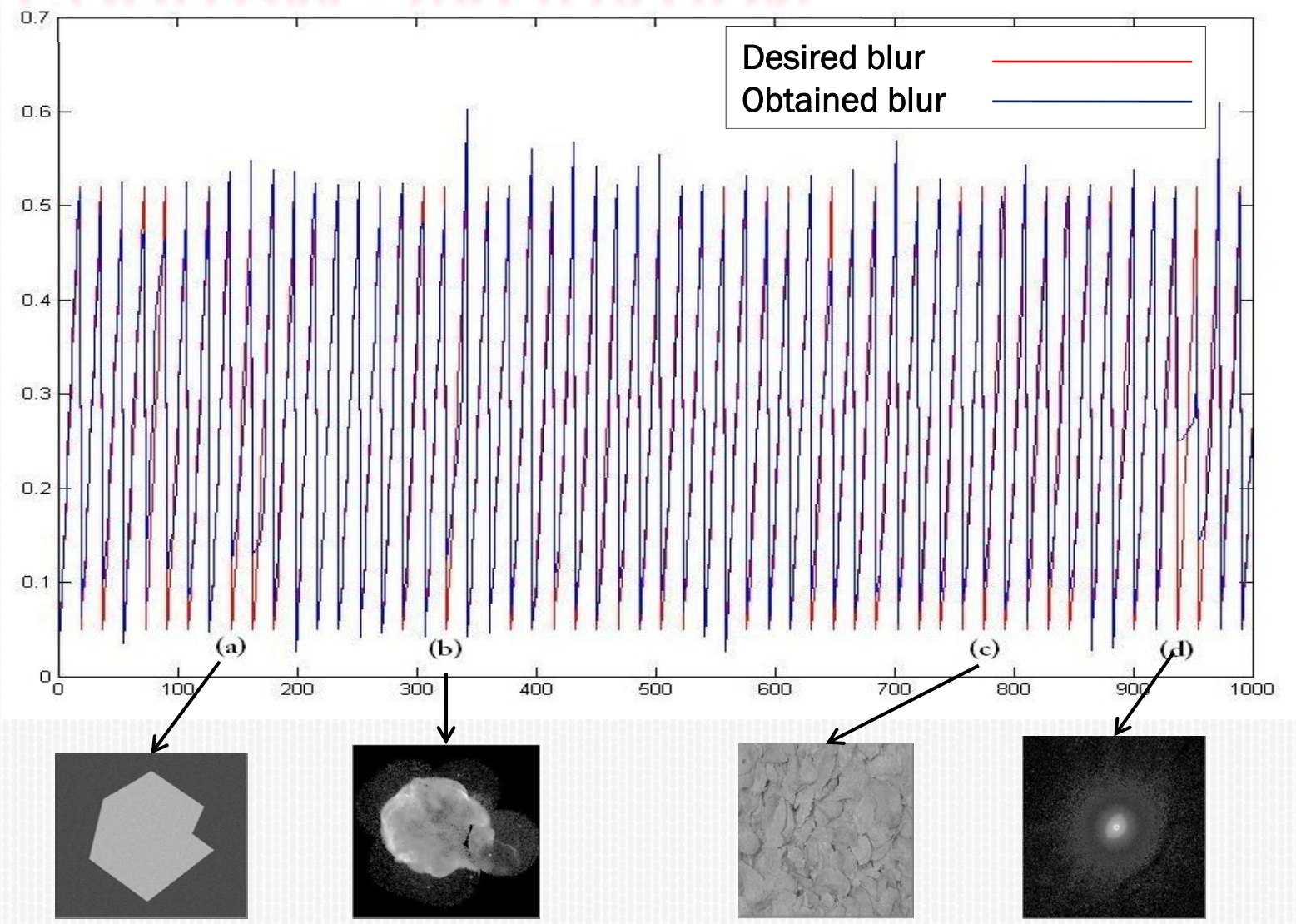
## Training for the prediction network

- Input to the network:
  - Image statistics
  - Initial blur, fractional order, resulting blur

## Performance Validation

- Feed in the training set (seen samples) and compute the deviation of the predicted value from the desired value.

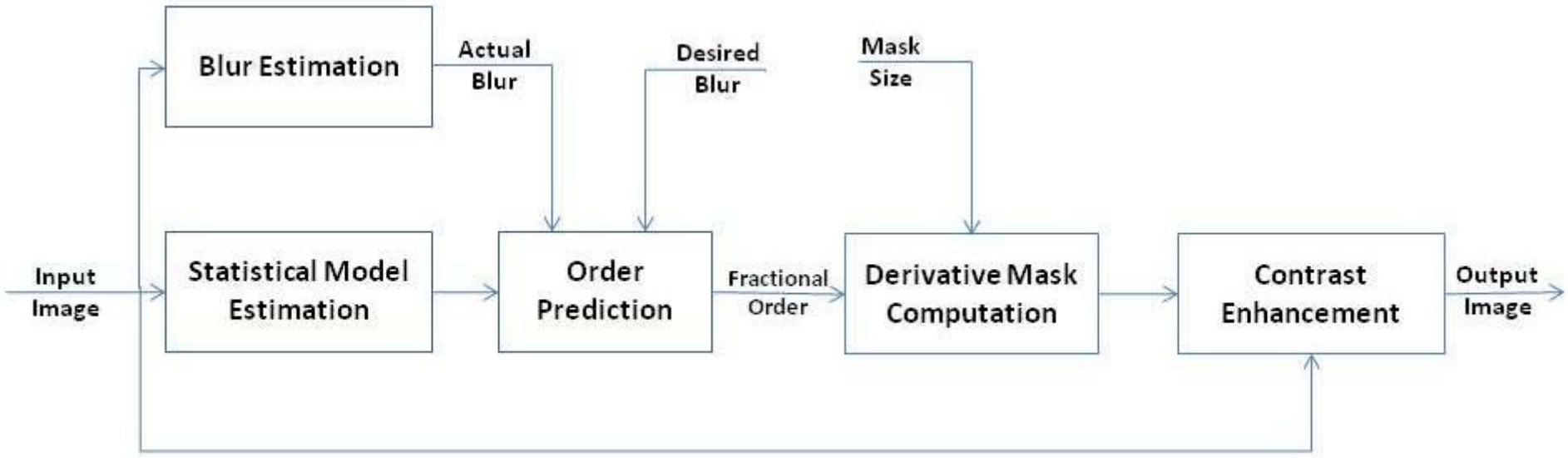
# ALGORITHM - VALIDATION



# IMAGE CONTRAST ENHANCEMENT

- Given any arbitrary image, compute
  - first four order statistics
  - the original blur
  - the desired blur
- Use trained RBF network to predict r.
- For this r,
  - compute the multi-directional mask
  - convolve the image
  - compute the resulting blur value
  - compare with desired blur reduction

# SCHEMATIC DIAGRAM



# RESULTS

Original
Blur=0.2575

Desired
Blur=0.1803

Enhanced
Blur=0.1859, r=0.35

# RESULTS

Original
Blur=0.2685

Desired
Blur=0.1880

Enhanced
Blur=0.2002, r=0.36

# RESULTS

Original
Blur=0.2541

Desired
Blur=0.1780

Enhanced
Blur=0.1771, r=0.346

# SUMMARY

## Strengths of the proposed system

- Controllability in respect of blur reduction
- Automated prediction of the fractional order

## Future work

- Improve the blur metric
- Local adaptation of filter
- Extend to color

*Thank You*